NON-LINEAR MODELS FOR GROWTH CURVES OF TRITICALE PLANTS UNDER IRRIGATION CONDITIONS

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ABSTRACT

The present work aimed to model the growth curves of some triticale cultivars with respect to their dry weight-age relationships and to determine a suitable non-linear model explaining their growth curve. For this purpose five different non-linear models were used to define growth curves of triticale plants, namely Gompertz, Logistic, Morgan-Mercer-Flodin, Weibull and Richards. The coefficients of determination for Richards's model were 0.996 (for Tatlıcak 97), 0.994 (for Melez 2001) and 0.997 (for Mikham 2001). Considering model selection criteria, Richards and Weibull models explained triticale growth better than Gompertz, Logistic, and Morgan-Mercer-Flodin. Richards and Weibull models seemed to be suitable models explaining triticale growth.
Key Words: Growth models, comparison criteria, dry matter weight, triticale

INTRODUCTION

Growth is defined as in increase the weight, number and length of cells (Efe, 1990). In general, nonlinear growth functions have been used to describe the dry weight-age curve in plants. These functions include Monomolecular autocatalytic, Exponential, Gompertz, Logistic, Richards, Morgan Mercer Flodin (MMF) and Weibull growth models (Seber and Wild, 1989; Keziol, 1986). Karadavut et. al. (2005) evaluated some growth models for describing dry weights of wheat.

The size of an individual character changes during growing and due to the differential growth of the particular body parts, the shape of an organism (its proportions) changes as well (Şengül and Kiraz, 2005). Unfortunately, one cannot measure continuously most of these growth processes. Therefore, it is preferable to model measurements by mathematical functions. This gives us the opportunity to interpolate to non-observed intervals (Şengül and Kiraz, 2005). Measurements of growth can be analyzed with respect to time (age) or to dry weight.

Growth trend defines periodic changes in the underlying characteristic. This change is affected by some environmental factors namely temperature, fertilizer pattern, and diseases etc., along with the growth.

There have been several studies undertaken toward the determination of growth trend in corns (*Zea mays* L.). Plant growth studies have also been conducted in wheat, forage plants and vegetable mostly. The literature on growth of plants define the age-dry weight relationship as a non-linear S-shaped function (Ratkowsky, 1986). Zahedi and Jenner (2003) defined the growth trend in wheat plants using the non-linear Gompertz model. On the other hand, there have been similar studies undertaken towards some plants using some non-linear models such as Logistic, Gompertz, Richards, Brody, Weibull and MMF (Mustears, 1989; Overman et. al., 1994; Zhang, 1997; Fekeduleng et. al., 1999; Schepers et.al., 2000; Willegas et.al., 2001; Damgaard et. al., 2002; Zahedi and Jenner, 2003; Fang and Gertner, 2004).

Growth parameters are important not only as selection criteria but also in terms of agronomic management techniques used during the production period and dry matter accumulation. This study aimed to estimate growth rate curves and their parameters using different growth models to determine the age-dry matter weight relationship in triticale plants.

MATERIALS AND METHODS

Tatlıcak 97, Melez 2001 and Mikham 2001 cultivars were used as genetic material in the study. These cultivars were developed and released by Bahri Dağdaş International Agricultural Research Institute. Plants were grown during the experiment period (2004-05 and 2005-06) in experimental areas of Bahri Dağdaş International Agricultural Research Institute. Three triticale cultivars were tested in a completely randomized block design with four replications. Each plot was 1.2 m x 5.0 m = 6 m^2. Planting distance was 0.20 m between rows and 0.6 m between plants. The plants were provided with fertilizer and water. Plants were fertilized using 150 kg/ha DAP (Di Ammonium Phosphate- 18% N, 46% P_2O_5) with sowing and 150 kg/ha Ammonium nitrate (21%) was used end of tillering. The experiment was undertaken from October to June in two years. Dry matter weights of triticale plants were recorded individually 10 days intervals from germination to maturation. Plants were irrigated 3 times (First year; 28 April, 21 Mai and 13 June and Second year 25 April, 18 Mai and 13 June)

In this research widely known non-linear growth models, Gompertz, Logistic, Morgan-Mercer-Flodin (MMF), Weibull and Richards, were fit to estimate the age-dry matter weight relationship. The mathematical relations of these models are as follows:

Gompertz : $y = a.\ e^{be^{\wedge} x}$ (1)

Logistic : $y = a / (1 + b.e^{cx})$, (2)

MMF : $y = (ab + c.x^d) / (b + x^d)$, (3)

Weibull : $y = a\text{-}be^{-cx^{\wedge}d}$ (4)

Richards : $y = 1 / (a + b.e^{cx})^d$, (5)

It relates dry weight (y) to age (x), where, a, b, c and d are explained following; a; the value of asymptote, b; value of plants growth beginning stage, c; net growth ratio, d; parameter at inflexion point. Non-linear regression procedure and Levenberg-Marquardt method of STATISTICA 6.0 statistical Program was used to estimate the parameters of all the models (Douglas and Donald, 1988).

Model evaluation: Examining the accuracy of the models used was tested by using coefficient of determination (R^2). The growth of triticale plants were variables on which the model predictions were compared with observed values. Five models were used for comparison. The statistical criteria used to compare the prediction (P_i) and observed (O_i) values for growth models were Eqs (6-8) as suggested by Mohanty ve Painuli (2004).

Modeling efficiency (ME):

$$ME = 1 - \frac{\sum_{i=1}^{i=n} (P_i - O_i)^2}{\sum_{i=1}^{i=n} (O_i - \bar{O})^2} \quad (6)$$

Root mean square error (RMSE):

$$RMSE = \frac{100}{\bar{O}} * \frac{\sqrt{\sum_{i=1}^{i=n} (P_i - O_i)^2}}{n} \quad (7)$$

The coefficient of residual mass (CRM):

$$CRM = \frac{\sum_{i=1}^{i=n} O_i - \sum_{i=1}^{i=n} P_i}{\sum_{i=1}^{i=n} O_i} \quad (8)$$

In the above equations, P_i ; prediction values, O_i ; observed values, n is the number of times heights of the growth were observed and $\bar{O}$; the mean observed values. The best model was selected with the highest R^2 and the lowest EF, RMSE, CRM and S_{yx} (Standard error of prediction) values.

RESULTS

Determination coefficients (R^2), ME, RMSE and CRM values are shown in Table 1. All models had considerably high R^2 values. The models may be ranked according to their R^2 values as MMF (0.968), Gompertz (0.979) and Logistic model (0.963) for Melez 2001, Weibull (0.996) and Richards (0.997) for Mikham 2001. The smallest values of comparison criteria were given respectively according to triticale cultivar. For S_{xy}; Melez 2001 cultivar had 15.92 (Gompertz), 19.17 (Logistic), 2.97 (Richards) and Melez 2001 cultivar had 16.86 (MMF) and 4.92 (Weibull). For ME; all growth models had become a value above 90%. For RMSE; Mikham 2001 cultivar had 139.88 (Gompertz) and 126.44 (Logistic) and Mikham 2001 cultivar had 151.18 (MMF), 117.18 (Weibull) and 83.47 (Richards). And for CRM; Mikham 2001 cultivar had 0.0676 (Logistics), 0.024 (Weibull), and 0.0011 (Richards), but Melez 2001 cultivar had 0.0406 (Gompertz). The growth curves of triticale cultivars are given in Figures 1, 2 and 3. As seen in these figures fit lines from all models are very close to the observed values.

Table 1. Coefficient of determination (R^2), parameter values and comparison criteria

Model	Cultivar	Comparison Criteria				
		R^2	S_{xy}	ME	RMSE	CRM
Gompertz	Tatlıcak 97	0,974	16,75	0,92	144,16	0,0428
	Melez 2001	**0,979**	**15,96**	0,92	**139,88**	**0,0406**
	Mikham 2001	0,969	17,44	0,90	149,74	0,0488
Logistic	Tatlıcak 97	0,954	20,11	0,91	138,31	0,0796
	Melez 2001	**0,963**	**19,17**	0,90	**126,44**	0,0698
	Mikham 2001	0,955	20,14	0,92	125,67	**0,0676**
MMF	Tatlıcak 97	0,953	21,59	0,95	181,14	0,0811
	Melez 2001	**0,968**	20,18	0,91	177,60	0,0796
	Mikham 2001	0,977	**16,85**	0,93	**151,18**	**0,0514**
Weibull	Tatlıcak 97	0,991	5,24	0,92	125,70	0,0030
	Melez 2001	0,991	5,20	0,91	119,62	0,0027
	Mikham 2001	**0,996**	**4,92**	0,91	**117,18**	**0,0024**
Richards	Tatlıcak 97	0,996	3,18	0,96	116,07	0,0016
	Melez 2001	0,994	**2,97**	0,96	98,11	0,0024
	Mikham 2001	**0,997**	2,30	0,94	**83,47**	**0,0011**

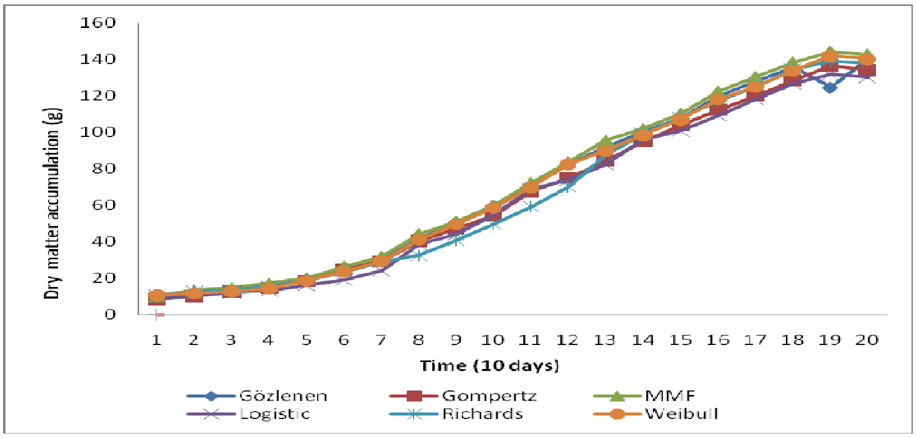

Figure 1. Growth curves for Cultivar Tatlıcak 97

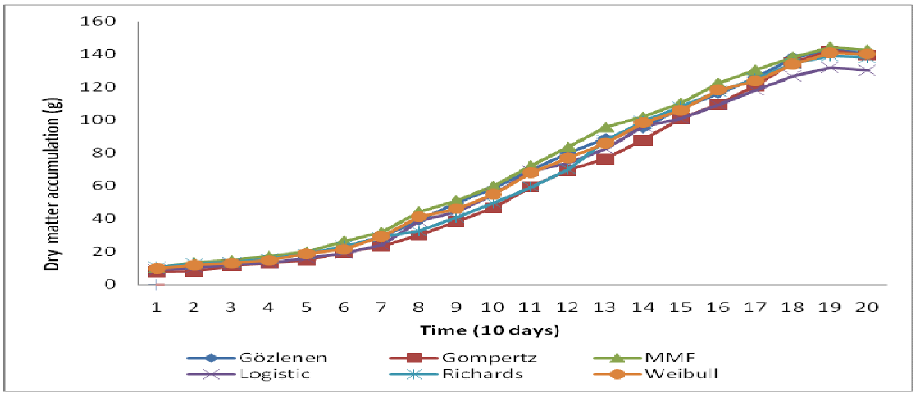

Figure 2. Growth curves for Cultivar Melez 2001

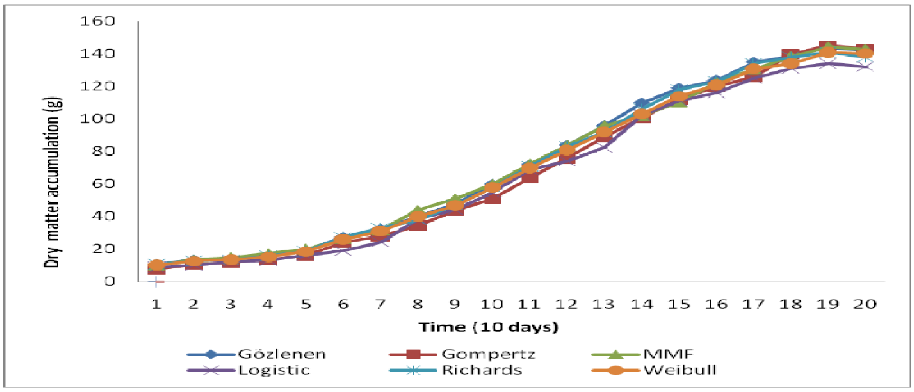

Figure 3. Growth curves for Cultivar Mikham 2001

DISCUSSION

The parameter of asymptotic growth (a) value is higher than other cultivars in Tatlıcak 97 (147.8) for the Gompertz, in Melez 2001 (135.2) for MMF and Mikham 2001 for Weibull (131.6) model. Considering Tatlıcak, Melez 2001 and Mikham 2001 cultivars, the value of plants growth beginning stage (b) parameters were found to be similar for Gompertz model. However, parameters obtained with the other four models are different according to cultivars. This difference may be related to the cultivar characteristics and environmental conditions on growth trend. Differences between growth rates of triticale growth have also been reported by Özer (2006). Environmental conditions might strongly affect on plant growth. Melez 2001 had the highest c (net growth ratio) values (0.023).

Gunartha (1995) have calculated the R^2 values for Logistic and Gompertz models for lettuce and they found 0.91 and 0.80 for the Logistic and Gompertz models, respectively. This research had higher values of R^2 for the Logistic model. Mustears (1989) used the Richards model for corn, alfalfa and trifolium and they found lower values of Mean Square Error for corn than those for alfalfa and trifolium, which ranged from 0.0796 to 0.1317, respectively. The higher values of Mean Square Error in trifolium could have been caused by high dry matter accumulation during the later stage of growth. In growing season, trifolium showed higher dry matter deposition, which represented 3,7% of dry weight while it amounted to only 1,8% in corn and 1,6% in alfalfa.

Many authors (Mustears, 1989; 1990; Overman et. al., 1994; Zhang, 1997; Fekeduleng et. al., 1999; Schepers et.al., 2000; Willegas et.al., 2001; Damgaard et. al., 2002; Zahedi and Jenner, 2003; Fang and Gertner, 2004) found similar results using the models outlined or different models. Models used here, in general, explain significant relationship between time and weight.

Richards model results, which had lowest RMSE, CRM and highest R^2 value, indicated that parameter A had meaning explaining triticale growth. Weibull seemed to be a better model explaining triticale regarding model selection criteria. Additionally the Richards and Weibull models performed well in this study.

The non-linear investigation of the growth process has some advantages not only mathematically explaining of growth but also estimating the relationship between agricultural practice's requirements and fresh weight, which plays a crucial role in plant growth. Furthermore, nonlinear estimation techniques may contribute to determining of the economic information and marketing strategies in plant-based enterprises.

Errors are the differences between observed and expected values and they are assumed to be zero. Standard error (S_{xy}) the least prediction method gives a good indication of the adequacy of the model. Among the tested models, the Richards model had the lowest, followed by the Weibull, Gompertz, MMF and Logistic models. From this study it could be said that the prediction obtained from Richards model with the lowest expected errors the other models. In this paper Tatlıcak 97 were found to have slightly higher S_{yx} values than those of Melez 2001 and Mikham 2001 in all models studied, except Logistic model. Goodness-of-fit was determined by ME statistics. ME statistics show a good efficiency for all triticale cultivars in all models.

LITERATURE CITED

Damgaard, C.; Weiner, J.; Nagashima, H. 2002. Modeling individual growth and competition in plant populations: growth curves of *Chjenopodium album* at two densities. Journal of Ecology. 90:666-671.

Douglas, M.B.; Donald, W.G. 1988. Non linear regression and its applications. John Wiley & Sons Inc. Canada.

Efe, E. 1990. Growth Curves. ÇÜ Institute of Basic and Applied Sciences. Department of Animal Science. Ph D. Thesis. Ankara.

Fang, S.; Gartner, G. 2004. Analysis of parameters of two growth models estimated using Bayesian methods and nonlinear regression. Cmsi.gre.ac.uk./conferences/iufro(proceedings)/ 5 October 2004 (retrieval date).

Fekeduleng, D.; Mac Siurtain, M.P.; Colbert, J.J. 1999. Parameter estimation of nonlinear growth models in forestry. Silva Fennica 33(4): 327-336.

Gunartha, L. 1995. Mechanistic models in lettuce growth. Ph. D. Thesis. University of Sidney. Australia.

Karadavut, U., Genç, A., Sinan, A., Karakoca, A., Palta, Ç., Aksoyak, Ş. 2005. Determination of growth curves in Dağdaş wheat plants. Statistical Researchs Symposium 9-13 Mayıs 2006. Belek-Antalya.

Keziol, J.A. 1986. An Introduction to multivariate growth curve analysis. Growth, 50:259-272.

Mohanty, M.; Painuli, D.K. 2004. Modeling rice seedling emergence and growth under tillage and residue management in a rice-wheat system on a vertisol in central India. Soil and Tillage Res. 76:167-174.

Mustears, H.J.W. 1989. A dynamic equation for plant interaction and application to yield-density-time relations. Annals of Botany. 64:521-531.

Overman, A.R.; Wilkinson, S.R.; Wilson, D.M. 1994. An extended model of forage grass response to applied nitrogen. Agronomy Journal, 86:617-620.

Özer, E. 2006. Effects of different sowing time and sowing densities under yield and yield components of triticale in Konya Province. SÜ Institute of Basic and Applied Sciences. Department of Agronomy. Ph D. Thesis. ,Konya.

Ratkowsky, D.A. 1986. Statistical properties of alternative parameterization of the von Bartalanffy growth curve. Can. J. Fish. Aquat. Sci. 43: 742-747.

Schepers, A.W.; Thibault, J.; Lacroix, C. 2000. Comparison of simple neural network and nonlinear regression models for description modeling of *Lactobacillus helveticus* growth in pH-controlled batch cultures. Enzyme Microb. Technol. 26:431-445.

Seber, G.A.F., Wild, C.J. 1989. Nonlinear Regression. John Wily&Sons.

Şengül, T., Kiraz, S. 2005. Non-linear models for growth curves in large white turkeys. Turk J. Vet. Animal Sci. 29:331-337.

Villegas, D.; Aparicio, N.; Blanco, R.; Royo, C. 2001. Biomass accumulation and main stem elongation of durum wheat grown under Mediterranean conditions. Ann. of Bot. 88:617-627.

Zahedi, M.; Jenner, C.F. 2003. Analysis of effects in wheat of high temperature on grain filling attributes estimated from mathematical models of grain filling. Journal of Agric. Sci. 141: 203-212.

Zhang, L. 1997. Cross-validation of non-linear growth functions for modeling tree height-diameter relationships. Annals of Botany. 79:251-257.